MATHEMATICS

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AIM POINT MATHEMATICS

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FUNCTION

& Their Properties

CONTENTS

THINGS TO REMEMBER

✷ Function or Mappings :

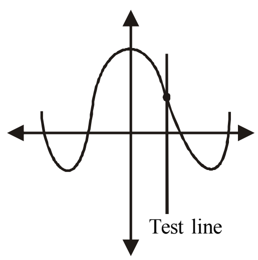
Let A and B be two non-empty sets. Then, a function f from set A to B is a rule which associates elements of set A to elementsof B such that all elements of set A are associated to elements of set B in unique way.

If f accociates $x \in$ A to y $\in$ B, then we say that y is the image of the element x and denote it by $f(x)$ and write as y = $f(x)$. The element x is called the pre-image of inverse image in B.

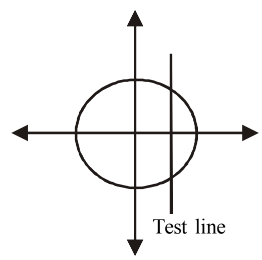
A function is denoted by $f : A \to B \, or \, A \xrightarrow{f} B$.

Difference between a Relation and a Function :

Geometrically, if we draw a vertical parallel line (VPL) ie, any line which is parallel to y-axis (x = a), then if this line intersects the graph of the expressionn in more than one point, then the expression is a relation else if it intersects at only one point, the expression is a function, eg.



(a) Function



(b) Relation

✷ Domain, Codomain and Range of a Function :

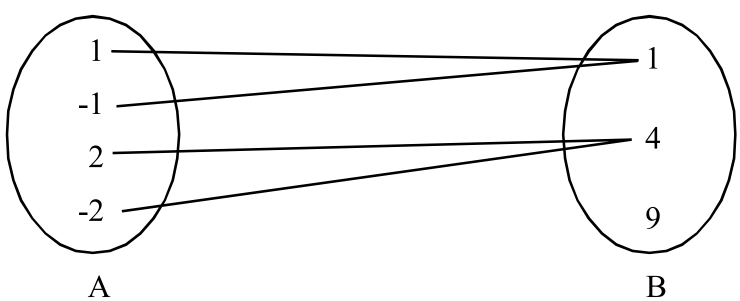
Lef f : A $\to$ B, then A is known as domain of f while B is known as codomain of f.

Also, set f(A) = {f(x) : $x \in$ A} is known as range of f.

Clearly, $f(A) \subseteq B$

eg, Let A = {1, -1, 2, -2}, B = {1, 4, 9}

$$f : A \xrightarrow{x^2} B \; ie, f(x) = x^2$$



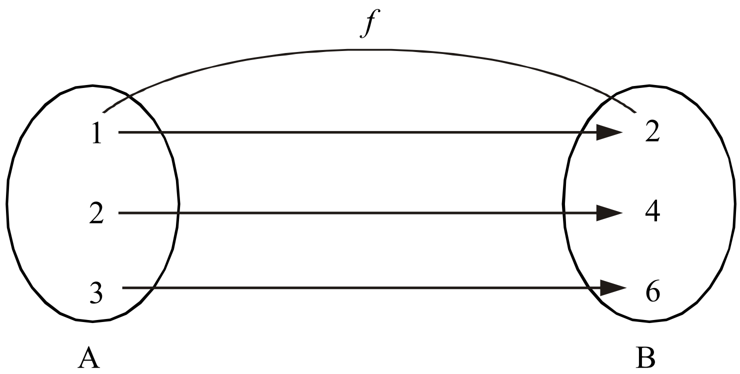
From the figure it is clear that domain of function = {1, -1, 2, -2} and range of function = {1, 4}

Also, codomain of function = {1, 4, 9}

✷ Different types of Function :

One-One Function (Injective)

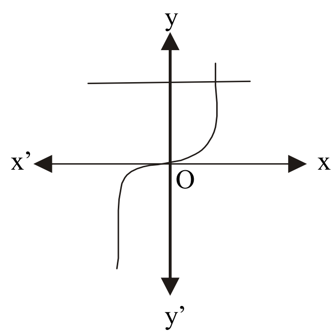
Let f : A $\to$ B. Then, f is said to be one-one function or an injective if different elements of A have different images in B.

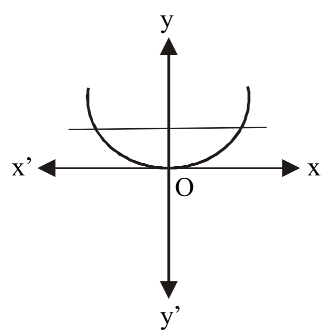


eg, Let A = {1, 2, 3} and B = {2, 4, 6}. Consider f : A → B, $f(x) = 2x$. Then, $f(1) = 2$, $f(2) = 4$ and $f(3) = 6$. Clearly, f is a one-one function from A to B such that different elements in A hvae different images in B.

Horizontal Parallel Lines Test (HPL Test) :

This is also a geometrical test to check whether a function is one-one or not.





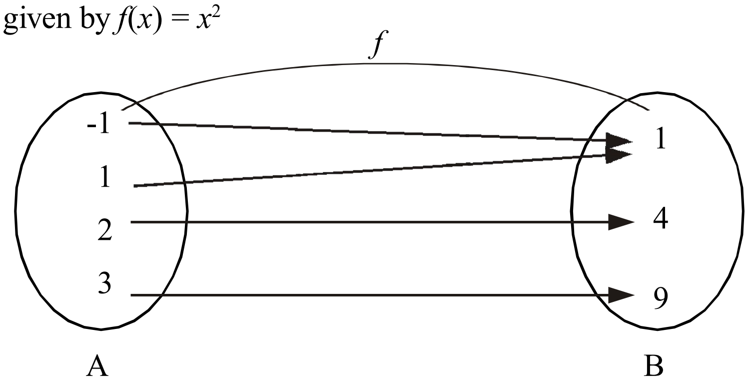
"If we draw a horizontal parallel line, ie, any line parallel to x-axis, then if this line intersects the graph of the function in at least two points, then the function in not one-one function, else, if it intersects at only one point, the function is one-one."

Many-One Function

Let f : A → B. If two or more than two elements of set A have the same image in B, then f is said to be many-one.

eg, Let A = {-1, 1, 2, 3} and B = {1, 4, 9}

where f : A → B given by $f(x) = x^2$

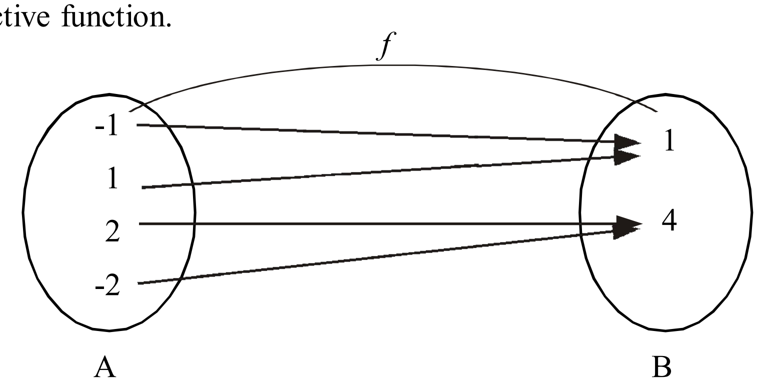


Here, $f(-1) = 1$; $f(1) = 1$; $f(2) = 4$ and $f(3) = 9$.

Clearly, two elements 1 and -1 have the same image 1 ∈ B. So, f is a many one function.

Onto Function (Surjective)

Let f : A → B. If every element in B has at least one pre-image in A, then f is said to be an onto function or surjective function.

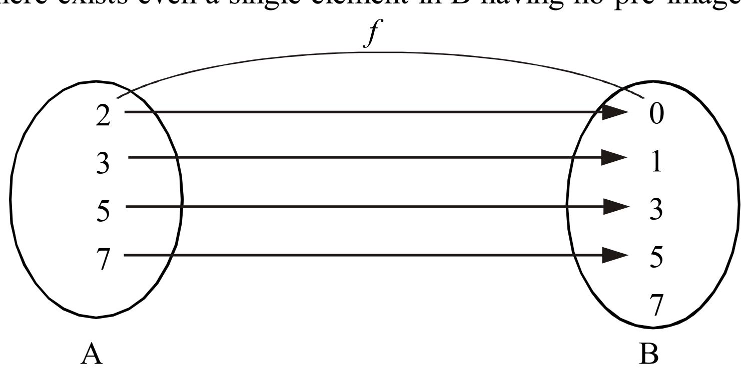


In this case, the range of f is a proper subset of codomain of *f*.

eg, Let A = {-1, 1, 2, -2}, B = {1, 4} and $f : A \to B$, be a function defined by $f(x) = x^2$. Then, *f* is onto because $f(A) = \{f(-1), f(1), f(2), f(-2)\} = \{1, 4\} = B$.

Into Function

Let f : A → B. If there exists even a single element in B having no pre-image in A, then f is said to be an into function.



eg, Let a {2, 3, 5, 7} and B = {0, 1, 3, 5, 7}. Consider $f : A \to B$; $f(x) = x - 2$. Then, $f(2) = 0$; $f(3)=1$; $f(5) = 3$ and $f(7) =5$.

Clearly, f is a function from A to B. Now, there exists an element $7 \in B$, having no pre-image in A. So, *f* is an into function.

Bijective Function (one-one onto Function)

A one-one and onto function is said to be bijetive. In other words, a function f : A → B is bijectie, If

(a) It is one-one ie, $f(x) = f(y) \Rightarrow x = y$ for all $x, y \in A$.

(b) It is onto *ie*, for all $y \in B$, there exists $x \in A$ such that $f(x) = y$.

Identity Function

Let A be a non-empty set. Then, the function, defined by $I_A : A \to A$, $I_A(x) = x$ for all $x \in A$, is called an identity funtion on A.

This is clearly a one-one onto function with domain A and range A.

Equal Function

Two Function *f* and *g* are said to be equal, written as $f = g$ if they have the same domain and they satify the condition $f(x) = g(x)$ for all x.

Inverse Function

Let f be a one-one onto function from A to B. Let y be an arbitrary element of B. Then, f being onto, there exists an element $x \in$ A such that $f(x)$ = y. Also, f being one-one, this this x must be unique. Thus, for each y. So, we may define a function,

$$f^{-1} : B \to A$$

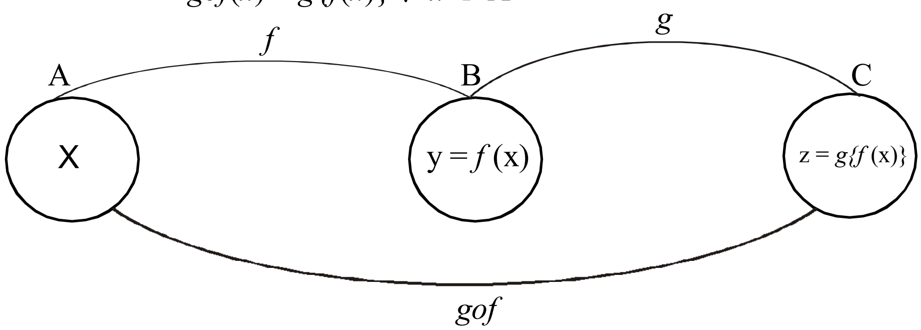
$$\therefore \quad f^{-1}(y) = x \quad \Leftrightarrow \quad f(x) = y$$

The above function f^{-1} is called the inverse of f.

Composition of Function

Let A, B and C be three non-empty sets. Let f : A → B and g : B → C be two mappings (or function) then gof : A → C. This function is called the product or composite of f and g given by

$$gof(x) = g\{f(x)\} \ \forall \ x \in A$$



gof exists if the range of f is a subset of domain of g. Similarly, fog exists if range of g is subset of domain of f.

Properties of Composite Function

(i) The composition of function is not commutative.

ie, $fog \neq gof$

(ii) The composition of function is associative.

ie, $fo(goh) = (fog)oh$

(iii) The composition of any function with the identity function is the functon itsself.

ie, If f : A → B, then $foI_A = I_B of = f$.

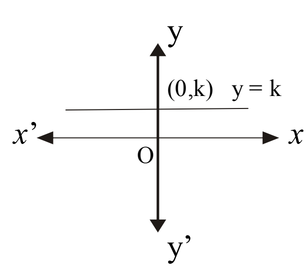
✷ **Classification of Function :**

Contant Function

A function which does not change as its parameters vary ie, the function whose rate of change is zero. In short, a constant function is a function that always gives of returns the same value.

OR

Let k be a constant, then function f(x) = k, $\forall \ x \in$ R is known as constant function.



Domain of $f(x)$ = R and Range of $f(x)$ = {k}

Polynomial Function

The function y = $f(x) = a_0x^n + a_1x^{n-1} + \ldots\ldots\ldots\ldots\ldots\ldots + a^n$, Where $a_0, a_1, a_2, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots a_n$ are real coefficients and n is a non-negative integer, is known as a polynomial function. If $a_0 \neq 0$, then degree of polynomial function is n.

Domain of $f(x)$ = R

On range varies from function to function.

Rational Function

If P(x) and Q(x) are polynfomial functions, Q(x) = 0, then function $f(x) = \frac{P(x)}{Q(x)}$ is known as rational function.

Domain fo $f(x) = R - \{x : Q(x) = 0\}$

and range varies from function to unction.

Irrational Function

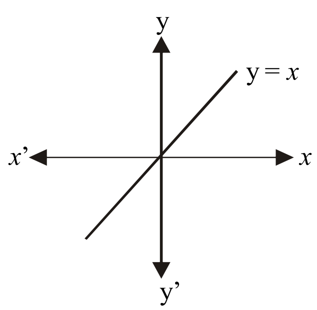
The function containing one or more terms having non-integral rational power of x are called irrational function.

eg, $$y = f(x) = \frac{5x^{3/2} - 7x^{1/2}}{x^{1/2} - 1}$$

Domain = varies from function to function

Identity Function

Function $f(x) = x, \ \forall \ x \in R$ is known as identity function. It is straight line passing through origin an having slope unity.



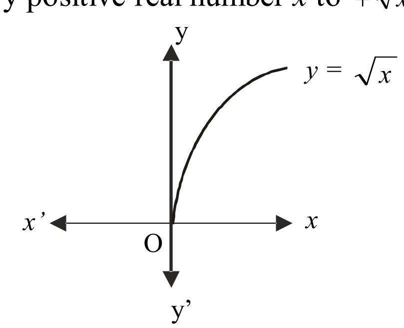
Domain of $f(x)$ = R

and Range of $f(x)$ = R

Square Root Function

The function that associates every positive real number x to $+\sqrt{x}$ is called the square root function, ie, $f(x) = +\sqrt{x}$

Rang of $f(x) = [0, \infty)$

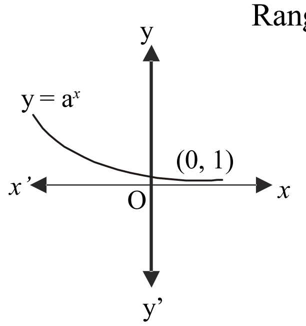


Exponential Function

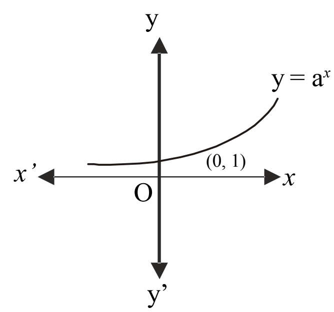
A function of the form $f(x) = a^x$, a is a positive real number, is an exponential function. The vlue of the function depends upon the value of a for $0 < a < 1$, function is decreasing and for $a > 1$, function is increasing.

Domain of $f(x) = R$

and Range of $f(x) = [0, \infty)$



$0 < a < 1$



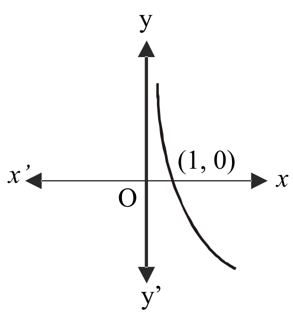
$a > 1$

Logarithmic Function

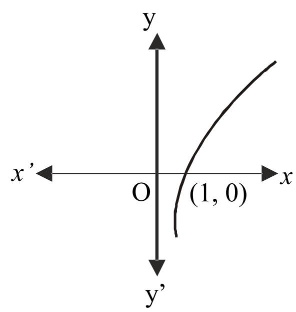
Function $f(x) = \log_a x$, $(x, a > 0)$ and $a \neq 1$, is knwon as logarithmic function.

Domain of $f(x) = (0, \infty)$

and Range of $f(x) = R$



$0 < a < 1$



$a < 1$

✷ **<u>Greatest Integer Function</u>**

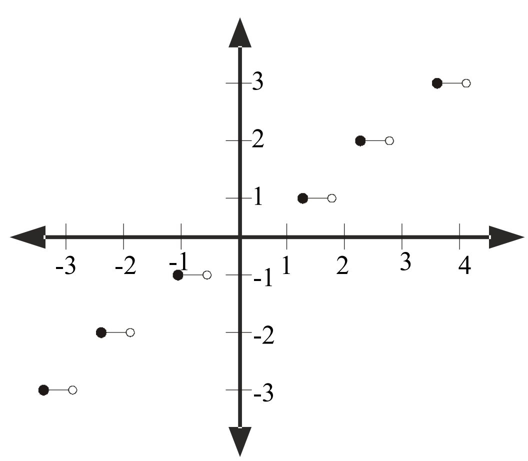
For any real number x, the greatest integer function $[x]$ is equal to greatest integer less than or equal to x.

In genreal, if n is an integer and x is any number satisfying $n \leq x < n + 1$, then $[x] = n$, it is also known as integral part function.

eg, If $2 \leq x < 3$, then $[x] = 2$

Domain = R

Range = I



Properties of Greatest integer Function

If n is an integer and x is any real number between n and n + 1, then

(i) $[-n] = -[n]$

(ii) $[x + n] = [x] + n$

(iii) $[-x] = -[x] - 1$, x is not an integer.

(iv) $[x + y] \geq [x] + [y]$

(v) $[x] > n \Rightarrow x \geq n + 1$

(vi) $[x] < n \Rightarrow x < n$

(vii) $[x + y] = [x] + [y + x - [x]]$, for all $x, y \in R$

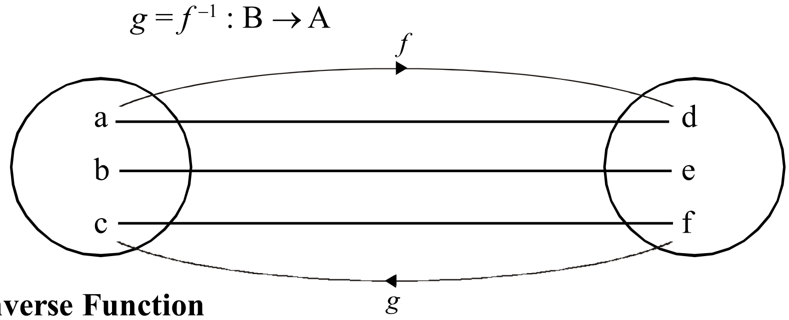
(viii) $[x] + \left[x + \frac{1}{n}\right] + \left[x + \frac{2}{n}\right] + \ldots\ldots\ldots\ldots + \left[x + \frac{n-1}{n}\right] = [nx]$,

$n \in N$

✱ <u>Inverse Function</u>

Let $f : A \to B$ is a bijective function, then there exists a unique fucntion $g : B \to A$ such that $f(x) = y \Leftrightarrow g(y) = x, \ \forall \ x \in A$ and $y \in B$, then g is called inverse function of f.

Hence, $g = f^{-1} : B \to A$



Properties of Inverse Function

(i) Inverse of bijective function is unique.

(ii) Inverse of bijective function is also bijective function.

(iii) If $f : A \to B$ is bijective function and $g : B \to A$ is inverse of f, then $fog = I_B$ and $gof = I_A$, where I_A and I_B are identity function of sets A and B respectively.

(iv) If $f : A \to B$ and $g : B \to A$ are two bijective function then $gof : A \to C$ is also bijective function and $(gof)^{-1} = f^{-1}og^{-1}$.

(v) $fog \neq gof$, but if $fog = gof$, then either $f^{-1} = g$ of $g^{-1} = f$ and $(fog)(x) = (gof)(x) = x$.

Note :

- There may exist some elements in set B which are not the images of any elements in set A.
- To each and every independent element in A there corresponds one and only one image in B.
- Every function is a relation but every reltion may of may not be a function.
- The number of function from a finite set A into finite set A into finite set B = $[n(B)]^{[n(A)]}$.
- If $x_1 \neq x_2 \Rightarrow f(x_1), f(x_2)$, for every $x_1, x_2 \in$ Domain, then f is one one of else many one.
- If $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$, for every $x_1, x_2 \in$ Domain, then f is one one of else many one.
- If the range of the function equals to the codomain of the function, then function is onto.
- The number of onto function that can be defined from a finite set A containing n elements onto a finite set B containing m elements = $2^n - m$.
- Inverse of bijective function is also bijective function.
- If the inverse of *f* exist, then *f* is called an inverible function, ie, A function *f* is invertible if and only if *f* is one one onto.
- $\log_b a = \frac{\log_c a}{\log_c b}$, where c is any constant, such that $c \in (0, \infty) - \{1\}$, a, b > 0.
- $a^{\log_c b} = b^{\log_c a}$
- If the inverse of f exists, then f is called an invertible function.